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In[1]:= Quit[];

In[2]:= f1[x_, y_] := -x + C1 ξ[x, y]^2 η[x, y] + C2 ξ[x, y]^2 + C3 ξ[x, y] +
C4 ξ[x, y] η[x, y]^2 + C5 η[x, y]^2 + C6 η[x, y] + C7 ξ[x, y] × η[x, y] + C8;
f2[x_, y_] := -y + D1 ξ[x, y]^2 η[x, y] + D2 ξ[x, y]^2 + D3 ξ[x, y] +
D4 ξ[x, y] η[x, y]^2 + D5 η[x, y]^2 + D6 η[x, y] + D7 ξ[x, y] × η[x, y] + D8;

In[3]:= D[f1[x, y], x]

Out[3]= 
$$-1 + C_3 \xi^{(1,0)}[x, y] + 2 C_2 \xi[x, y] \xi^{(1,0)}[x, y] + C_7 \eta[x, y] \xi^{(1,0)}[x, y] +$$


$$2 C_1 \xi[x, y] \times \eta[x, y] \xi^{(1,0)}[x, y] + C_4 \eta[x, y]^2 \xi^{(1,0)}[x, y] +$$


$$C_6 \eta^{(1,0)}[x, y] + C_7 \xi[x, y] \eta^{(1,0)}[x, y] + C_1 \xi[x, y]^2 \eta^{(1,0)}[x, y] +$$


$$2 C_5 \eta[x, y] \eta^{(1,0)}[x, y] + 2 C_4 \xi[x, y] \times \eta[x, y] \eta^{(1,0)}[x, y]$$


In[4]:= D[f2[x, y], x]

Out[4]= 
$$D_3 \xi^{(1,0)}[x, y] + 2 D_2 \xi[x, y] \xi^{(1,0)}[x, y] + D_7 \eta[x, y] \xi^{(1,0)}[x, y] +$$


$$2 D_1 \xi[x, y] \times \eta[x, y] \xi^{(1,0)}[x, y] + D_4 \eta[x, y]^2 \xi^{(1,0)}[x, y] +$$


$$D_6 \eta^{(1,0)}[x, y] + D_7 \xi[x, y] \eta^{(1,0)}[x, y] + D_1 \xi[x, y]^2 \eta^{(1,0)}[x, y] +$$


$$2 D_5 \eta[x, y] \eta^{(1,0)}[x, y] + 2 D_4 \xi[x, y] \times \eta[x, y] \eta^{(1,0)}[x, y]$$


In[5]:= D[f1[x, y], y]

Out[5]= 
$$C_3 \xi^{(0,1)}[x, y] + 2 C_2 \xi[x, y] \xi^{(0,1)}[x, y] + C_7 \eta[x, y] \xi^{(0,1)}[x, y] +$$


$$2 C_1 \xi[x, y] \times \eta[x, y] \xi^{(0,1)}[x, y] + C_4 \eta[x, y]^2 \xi^{(0,1)}[x, y] +$$


$$C_6 \eta^{(0,1)}[x, y] + C_7 \xi[x, y] \eta^{(0,1)}[x, y] + C_1 \xi[x, y]^2 \eta^{(0,1)}[x, y] +$$


$$2 C_5 \eta[x, y] \eta^{(0,1)}[x, y] + 2 C_4 \xi[x, y] \times \eta[x, y] \eta^{(0,1)}[x, y]$$


In[6]:= D[f2[x, y], y]

Out[6]= 
$$-1 + D_3 \xi^{(0,1)}[x, y] + 2 D_2 \xi[x, y] \xi^{(0,1)}[x, y] + D_7 \eta[x, y] \xi^{(0,1)}[x, y] +$$


$$2 D_1 \xi[x, y] \times \eta[x, y] \xi^{(0,1)}[x, y] + D_4 \eta[x, y]^2 \xi^{(0,1)}[x, y] +$$


$$D_6 \eta^{(0,1)}[x, y] + D_7 \xi[x, y] \eta^{(0,1)}[x, y] + D_1 \xi[x, y]^2 \eta^{(0,1)}[x, y] +$$


$$2 D_5 \eta[x, y] \eta^{(0,1)}[x, y] + 2 D_4 \xi[x, y] \times \eta[x, y] \eta^{(0,1)}[x, y]$$


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In[8]:= (* to get dξ/dx and dη/dx *)
Solve[D[f1[x, y], x] == 0 && D[f2[x, y], x] == 0, {ξ^(1,0)[x, y], η^(1,0)[x, y]}]

Out[8]= {{ξ^(1,0)[x, y] → -((D6 + D7 ξ[x, y] + D1 ξ[x, y]^2 + 2 D5 η[x, y] + 2 D4 ξ[x, y] × η[x, y]) / (-D6 + D7 ξ[x, y] + D1 ξ[x, y]^2 + 2 D5 η[x, y] + 2 D4 ξ[x, y] × η[x, y]) + (C3 + 2 C2 ξ[x, y] + C7 η[x, y] + 2 C1 ξ[x, y] × η[x, y] + C4 η[x, y]^2) + (C6 + C7 ξ[x, y] + C1 ξ[x, y]^2 + 2 C5 η[x, y] + 2 C4 ξ[x, y] × η[x, y]) + (D3 + 2 D2 ξ[x, y] + D7 η[x, y] + 2 D1 ξ[x, y] × η[x, y] + D4 η[x, y]^2))), η^(1,0)[x, y] → -((-D3 - 2 D2 ξ[x, y] - D7 η[x, y] - 2 D1 ξ[x, y] × η[x, y] - D4 η[x, y]^2) / (C6 D3 - C3 D6 + 2 C6 D2 ξ[x, y] + C7 D3 ξ[x, y] - 2 C2 D6 ξ[x, y] - C3 D7 ξ[x, y] - C3 D1 ξ[x, y]^2 + 2 C7 D2 ξ[x, y]^2 + C1 D3 ξ[x, y]^2 - 2 C2 D7 ξ[x, y]^2 - 2 C2 D1 ξ[x, y]^3 + 2 C1 D2 ξ[x, y]^3 + 2 C5 D3 η[x, y] - 2 C3 D5 η[x, y] - C7 D6 η[x, y] + C6 D7 η[x, y] + 2 C6 D1 ξ[x, y] × η[x, y] + 4 C5 D2 ξ[x, y] × η[x, y] + 2 C4 D3 ξ[x, y] × η[x, y] - 2 C3 D4 ξ[x, y] × η[x, y] - 4 C2 D5 ξ[x, y] × η[x, y] - 2 C1 D6 ξ[x, y] × η[x, y] + C7 D1 ξ[x, y]^2 η[x, y] + 4 C4 D2 ξ[x, y]^2 η[x, y] - 4 C2 D4 ξ[x, y]^2 η[x, y] - C1 D7 ξ[x, y]^2 η[x, y] + C6 D4 η[x, y]^2 - 2 C7 D5 η[x, y]^2 - C4 D6 η[x, y]^2 + 2 C5 D7 η[x, y]^2 + 4 C5 D1 ξ[x, y] η[x, y]^2 - C7 D4 ξ[x, y] η[x, y]^2 - 4 C1 D5 ξ[x, y] η[x, y]^2 + C4 D7 ξ[x, y] η[x, y]^2 + 3 C4 D1 ξ[x, y]^2 η[x, y]^2 - 3 C1 D4 ξ[x, y]^2 η[x, y]^2 + 2 C5 D4 η[x, y]^3 - 2 C4 D5 η[x, y]^3))}}}

In[9]:= (* to get dξ/dy and dη/dy *)
Solve[D[f1[x, y], y] == 0 && D[f2[x, y], y] == 0, {ξ^(0,1)[x, y], η^(0,1)[x, y]}]

Out[9]= {{ξ^(0,1)[x, y] → -((-C6 - C7 ξ[x, y] - C1 ξ[x, y]^2 - 2 C5 η[x, y] - 2 C4 ξ[x, y] × η[x, y]) / (-D6 + D7 ξ[x, y] + D1 ξ[x, y]^2 + 2 D5 η[x, y] + 2 D4 ξ[x, y] × η[x, y]) + (C3 + 2 C2 ξ[x, y] + C7 η[x, y] + 2 C1 ξ[x, y] × η[x, y] + C4 η[x, y]^2) + (C6 + C7 ξ[x, y] + C1 ξ[x, y]^2 + 2 C5 η[x, y] + 2 C4 ξ[x, y] × η[x, y]) + (D3 + 2 D2 ξ[x, y] + D7 η[x, y] + 2 D1 ξ[x, y] × η[x, y] + D4 η[x, y]^2))), η^(0,1)[x, y] → -((C3 + 2 C2 ξ[x, y] + C7 η[x, y] + 2 C1 ξ[x, y] × η[x, y] + C4 η[x, y]^2) / (C6 D3 - C3 D6 + 2 C6 D2 ξ[x, y] + C7 D3 ξ[x, y] - 2 C2 D6 ξ[x, y] - C3 D7 ξ[x, y] - C3 D1 ξ[x, y]^2 + 2 C7 D2 ξ[x, y]^2 + C1 D3 ξ[x, y]^2 - 2 C2 D7 ξ[x, y]^2 - 2 C2 D1 ξ[x, y]^3 + 2 C1 D2 ξ[x, y]^3 + 2 C5 D3 η[x, y] - 2 C3 D5 η[x, y] - C7 D6 η[x, y] + C6 D7 η[x, y] + 2 C6 D1 ξ[x, y] × η[x, y] + 4 C5 D2 ξ[x, y] × η[x, y] + 2 C4 D3 ξ[x, y] × η[x, y] - 2 C3 D4 ξ[x, y] × η[x, y] - 4 C2 D5 ξ[x, y] × η[x, y] - 2 C1 D6 ξ[x, y] × η[x, y] + C7 D1 ξ[x, y]^2 η[x, y] + 4 C4 D2 ξ[x, y]^2 η[x, y] - 4 C2 D4 ξ[x, y]^2 η[x, y] - C1 D7 ξ[x, y]^2 η[x, y] + C6 D4 η[x, y]^2 - 2 C7 D5 η[x, y]^2 - C4 D6 η[x, y]^2 + 2 C5 D7 η[x, y]^2 + 4 C5 D1 ξ[x, y] η[x, y]^2 - C7 D4 ξ[x, y] η[x, y]^2 - 4 C1 D5 ξ[x, y] η[x, y]^2 + C4 D7 ξ[x, y] η[x, y]^2 + 3 C4 D1 ξ[x, y]^2 η[x, y]^2 - 3 C1 D4 ξ[x, y]^2 η[x, y]^2 + 2 C5 D4 η[x, y]^3 - 2 C4 D5 η[x, y]^3))}}}

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